# Three-Way Entanglement and Three-Qubit Phase Gate Based on a Coherent Six-Level Atomic System

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## Abstract

We analyze the nonlinear optical response of a six-level atomic system under a configuration of electromagnetically induced transparency. The giant fifth-order nonlinearity generated in such a system with a relatively large cross-phase modulation effect can produce efficient three-way entanglement and may be used for realizing a three-qubit quantum phase gate. We demonstrate that such phase gate can be transferred to a Toffoli gate, facilitating practical applications in quantum information and computation.

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Photons are considered as promising candidates for carrying quantum information because of their high propagating speed and negligible decoherence[1]. Many proposals have come up for efficiently implementing all-optical quantum information processing and quantum computation, some of which are based on linear optics, and others are considered from nonlinear optical processes. As is well known, Kerr nonlinearity is crucial for producing photon-photon entanglement and for realizing two-qubit optical quantum gates. Similarly, higher-order optical nonlinearities can be used to produce an $N$-way ($N \geq 3$) entanglement and realize a multi-qubit quantum gate. However, optical quantum gates can not be efficiently implemented based on a conventional optical medium. The reason is that either the optical nonlinearity produced in such medium is very weak, or there is a very large optical absorption when working near resonant regime where nonlinear effect is strong.

In recent years, much attention has been paid to the study of electromagnetically induced transparency (EIT) in resonant atomic systems[2, 3]. By means of the effect of quantum coherence and interference induced by a control field, the absorption of a weak probe field tuned to a strong one-photon resonance can be largely cancelled and hence an initially highly opaque optical medium becomes transparent. The wave propagation in a resonant optical medium with EIT configuration possesses many striking features. One of them is the significant reduction of the group velocity of the probe pulse. Another is the giant enhancement of the Kerr nonlinearity of the optical medium[4, 5]. Several suggestions for obtaining enhanced Kerr nonlinearity and a related large cross-phase modulation (CPM) by using the EIT effect have been proposed , including "N" configuration[6, 7], chain-$\Lambda$ configuration[8], tripod configuration[9], and symmetric six-level configuration[10]. Based on the enhanced Kerr nonlinearity, two-qubit entanglement with photons and atoms [11, 12, 13, 14, 15, 16] has been investigated and all-optical two-qubit quantum phase gate (QPG)[17, 18, 19, 20] has also been constructed by using different schemes recently. However, as far as we know, up to now only a few works[21] have studied higher-order, especially the fifth-order optical nonlinearity, and its applications to multi-photon entanglement and optical phase gates under practice EIT configurations.

In this work, we investigate the nonlinear optical response and possible three-way entanglement and three-qubit phase gates based on a coherent six-level atomic system under an asymmetric EIT configuration. Our study shows that, due to quantum interference, fifth-order nonlinearity in such system can be largely enhanced with a vanishing linear and

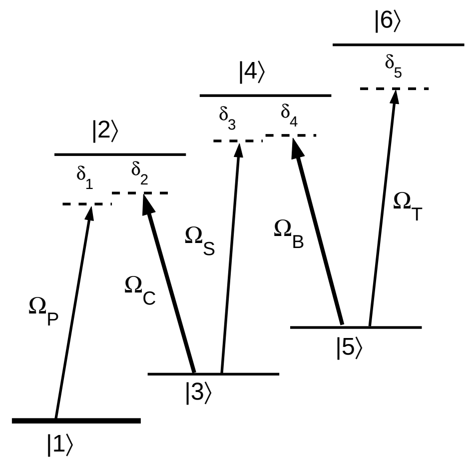


FIG. 1: The energy-level diagram and excitation scheme of a life-time broadened six-level atomic system interacting with two strong, cw control fields of Rabi frequencies $\Omega_C$ and $\Omega_B$, and three weak, pulsed (probe, signal, and trigger) fields of Rabi frequencies $\Omega_P$, $\Omega_S$ and $\Omega_T$.

third-order nonlinear effect. Using this property the system can be used to produce efficient three-way entanglement among three optical (probe, signal, and trigger) fields. We then explore the possibility of employing an enhanced CPM effect to devise a mechanism of polarization three-qubit quantum phase gate (QPG). The three-qubit QPG proposed here is rather robust, and can be easily transferred to a universal three-qubit Toffoli gate. Although a Toffli gate can be constructed by other basic quantum gates, its realization in a more compact way is needed to dramatically reduce the number of qubit and manipulations that are required to perform a given task. Although some studies of constructing Toffoli gate with different systems[22, 23, 24, 25] exist, our work presented here is for the first time a practical realization of Toffoli gate in an all-optical way.

We start with considering a life-time broadened atomic system, where atoms with six levels (three ground state levels $|1\rangle$, $|3\rangle$, $|5\rangle$, and three excited state levels $|2\rangle$, $|4\rangle$, $|6\rangle$) interact with five laser fields (see Fig.1). Such configuration can be realized in Zeeman-splitted alkali atoms (e.g., the $D1$ line in $^{23}$Na or $^{87}$Rb gas). We assume that the transitions from $|2\rangle \leftrightarrow |3\rangle$ and $|4\rangle \leftrightarrow |5\rangle$ are driven by two strong, continuous-wave (cw) laser control fields, with Rabi frequencies $\Omega_C$ and $\Omega_B$, respectively. The transitions from $|1\rangle \leftrightarrow |2\rangle$, $|3\rangle \leftrightarrow |4\rangle$, and $|5\rangle \leftrightarrow |6\rangle$ are driven by three weak, pulsed laser fields, called probe field (with Rabi frequency $\Omega_P$) signal field (with Rabi frequency $\Omega_S$) and trigger field (with Rabi

frequency $\Omega_T$), respectively. Here the Rabi frequencies associated with the laser fields that drive the atomic transitions are defined as $\Omega_k = -D_{ij}\mathcal{E}_k/\hbar$, where $\mathcal{E}_k$ denotes the $k$th electric field envelope and $D_{ij}$ is the relevant electric-dipole matrix element related to the transition $|i\rangle \leftrightarrow |j\rangle$. The detunings $\delta_i$ are defined as $\delta_1 = (E_2 - E_1)/\hbar - \omega_P$, $\delta_2 = (E_2 - E_3)/\hbar - \omega_C$, $\delta_3 = (E_4 - E_3)/\hbar - \omega_S$, $\delta_4 = (E_4 - E_5)/\hbar - \omega_B$, and $\delta_5 = (E_6 - E_5)/\hbar - \omega_T$, where $E_i$ ($i$=1,...,6) is the energy of the level $|i\rangle$ and $\omega_j$ ($j$=P, C, S, B, and T) is the frequency of the laser field with the Rabi frequency $\Omega_j$. The evolution equations for the probability amplitudes $a_i(t)$ of the atomic state $|\psi(t)\rangle = \sum_{i=1}^{6} a_i(t)|i\rangle$ are

$$\dot{a}_1 = -\frac{\Gamma_1}{2}a_1 - i\Omega_P^* a_2, \tag{1a}$$

$$\dot{a}_2 = -(\frac{\Gamma_2}{2} + i\delta_1)a_2 - i\Omega_P a_1 - i\Omega_C a_3, \tag{1b}$$

$$\dot{a}_3 = -(\frac{\Gamma_3}{2} + i\delta_{12})a_3 - i\Omega_C^* a_2 - i\Omega_S^* a_4, \tag{1c}$$

$$\dot{a}_4 = -(\frac{\Gamma_4}{2} + i\delta_{13})a_4 - i\Omega_S a_3 - i\Omega_B a_5, \tag{1d}$$

$$\dot{a}_5 = -(\frac{\Gamma_5}{2} + i\delta_{14})a_5 - i\Omega_B^* a_4 - i\Omega_T^* a_6, \tag{1e}$$

$$\dot{a}_6 = -(\frac{\Gamma_6}{2} + i\delta_{15})a_6 - i\Omega_T a_5, \tag{1f}$$

where $\delta_{12} = \delta_1 - \delta_2$, $\delta_{13} = \delta_{12} + \delta_3$, $\delta_{14} = \delta_{13} - \delta_4$, and $\delta_{15} = \delta_{14} + \delta_5$. $\Gamma_i$ denotes the decay rate for the atomic level $|i\rangle$. For the excited state levels ($|2\rangle$, $|4\rangle$, and $|6\rangle$) these rates describe the total spontaneous decay rates, while for the ground state levels ($|1\rangle$, $|3\rangle$, and $|5\rangle$) the associated decay rates describe dephasing processes.

For solving Eq. (1) we assume that the typical temporal duration of the probe, signal, and trigger fields is long enough so that a steady state approximation can be employed. The system's initial state is assumed to be the ground state $|1\rangle$. When the intensity of the probe, signal, and trigger is much weaker than the intensity of both coupling fields, the population in the ground states $|1\rangle$ is not depleted even when the system reaches the steady state, i.e. $a_0 \approx 1$. We solve Eq. (1) under these consideration and obtain the following expressions for the susceptibilities of three weak fields

$$\chi_P \simeq \chi_P^{(1)} + \chi_{PS}^{(3)}|E_S|^2 + \chi_{PT}^{(3)}|E_T|^2 + \chi_{PST}^{(5)}|E_S|^2|E_T|^2, \tag{2a}$$

$$\chi_S \simeq \chi_{SP}^{(3)}|E_P|^2 + \chi_{SPT}^{(5)}|E_P|^2|E_T|^2, \tag{2b}$$

$$\chi_T \simeq \chi_{TPS}^{(5)}|E_P|^2|E_S|^2, \tag{2c}$$

with

$$\chi_P^{(1)} = \frac{N_a|D_{12}|^2}{\hbar\epsilon_0}\frac{d_3}{d_2d_3-|\Omega_C|^2}, \tag{3a}$$

$$\chi_{PS}^{(3)} = -\frac{N_a|D_{12}|^2|D_{34}|^2}{\hbar^3\epsilon_0}\frac{d_5}{(d_4d_5-|\Omega_B|^2)(d_2d_3-|\Omega_C|^2)}, \tag{3b}$$

$$\chi_{PT}^{(3)} = -\frac{N_a|D_{12}|^2|D_{56}|^2}{\hbar^3\epsilon_0}\frac{d_3d_4}{d_6(d_4d_5-|\Omega_B|^2)(d_2d_3-|\Omega_C|^2)}, \tag{3c}$$

$$\chi_{PST}^{(5)} = \frac{N_a|D_{12}|^2|D_{34}|^2|D_{56}|^2}{\hbar^5\epsilon_0}\frac{1}{d_6(d_4d_5-|\Omega_B|^2)(d_2d_3-|\Omega_C|^2)}, \tag{3d}$$

$$\chi_{SP}^{(3)} = \frac{N_a|D_{12}|^2|D_{34}|^2}{\hbar^3\epsilon_0}\frac{d_5|\Omega_C|^2}{(d_4d_5-|\Omega_B|^2)|d_2d_3-|\Omega_C|^2|^2}, \tag{3e}$$

$$\begin{aligned}\chi_{SPT}^{(5)} = &-\frac{N_a|D_{12}|^2|D_{34}|^2|D_{56}|^2}{\hbar^5\epsilon_0}\left[\frac{|\Omega_C|^2}{d_6(d_4d_5-|\Omega_B|^2)|d_2d_3-|\Omega_C|^2|^2}\right.\\ &\left.+\frac{d_4^*d_5|\Omega_C|^2}{d_6^*|d_4d_5-|\Omega_B|^2|^2|d_2d_3-|\Omega_C|^2|^2}\right],\end{aligned} \tag{3f}$$

$$\chi_{TPS}^{(5)} = \frac{N_a|D_{12}|^2|D_{34}|^2|D_{56}|^2}{\hbar^5\epsilon_0}\frac{|\Omega_B|^2|\Omega_C|^2}{d_6|d_4d_5-|\Omega_B|^2|^2|d_2d_3-|\Omega_C|^2|^2}. \tag{3g}$$

Here $\chi^{(1)}$, $\chi^{(3)}$, and $\chi^{(5)}$ denote the linear, third-order, and fifth-order susceptibilities corresponding each field, star denotes the complex conjugation, and $N_a$ is the density of the atomic gas. We have defined $d_2 = \delta_1 - i\Gamma_2/2$ ,$d_3 = \delta_{12} - i\Gamma_3/2$, $d_4 = \delta_{13} - i\Gamma_4/2$, $d_5 = \delta_{14} - i\Gamma_5/2$, and $d_6 = \delta_{15} - i\Gamma_6/2$.

Above results show that the nonlinear susceptibilities associated with CPM can be largely enhanced. This can be seen from Eq. (3) that, under the conditions $d_3 \approx d_5 \approx 0$[26], the fifth-order susceptibilities remain and have comparably giant values while the linear and third-order susceptibilities being efficiently suppressed. Thus under such conditions the system provides only a fifth-order nonlinear effect. In addition, the imaginary parts of the linear and nonlinear susceptibilities given above are much smaller than their relevant real parts under the (EIT) condition $|\Omega_P|^2$, $|\Omega_S|^2$, and $|\Omega_T|^2 \ll |\Omega_C|^2, |\Omega_B|^2$, which results in quantum interferences between the states $|1\rangle \leftrightarrow |3\rangle$ and $|3\rangle \leftrightarrow |5\rangle$, making the population in the excited states be small thus very low absorption for the probe, signal, and trigger fields.

The results (2) and (3) enable one to asses the group velocities of the probe, signal, and trigger fields. As we know, group velocities have to be comparable and small in order to achieve an effective CPM effect[27]. Unlike the six-level scheme studied in[10], the present scheme is not symmetric and hence probe, signal, and trigger group velocities are generally not equal. Assuming working at the center of the transparency window for the probe and

signal fields, i. e. $\delta_{12} \approx \delta_{14} \approx 0$, and neglecting the dephasing rates $\Gamma_3$ and $\Gamma_5$, which are typically much smaller than all the other parameters, we obtain the expressions of group velocities from (2)-(3) for the the probe, signal, and trigger fields as

$$v_g^P \simeq \frac{2\hbar\epsilon_0 c|\Omega_C|^2|\Omega_B|^2}{N_a|D_{12}|^2\omega_P(|\Omega_B|^2+|\Omega_S|^2+|\Omega_T|^2\beta_1-|\Omega_S|^2|\Omega_T|^2\beta_2)}, \tag{4a}$$

$$v_g^S \simeq \frac{2\hbar\epsilon_0 c|\Omega_C|^2|\Omega_B|^2}{N_a|D_{34}|^2\omega_S|\Omega_P|^2(1+|\Omega_T|^2\beta)}, \tag{4b}$$

$$v_g^T \simeq \frac{2\hbar\epsilon_0 c|\Omega_C|^2|\Omega_B|^2}{N_a|D_{56}|^2\omega_T|\Omega_P|^2|\Omega_S|^2\beta}, \tag{4c}$$

with $\beta_1 = (\delta_3\delta_5+\Gamma^2/4)/(\delta_5^2+\Gamma^2/4)$, $\beta_2 = [(\delta_3\delta_5+\Gamma^2/4)(\delta_5^2+\Gamma^2/4)/|\Omega_B|^2+(\delta_1\delta_5+\Gamma^2/4)(\delta_5^2+\Gamma^2/4)/|\Omega_C|^2-(\delta_5^2-\Gamma^2/4)]/(\delta_5^2+\Gamma^2/4)^2$, and $\beta = (\delta_5^2-\Gamma^2/4)/(\delta_5^2+\Gamma^2/4)^2$. For simplicity for getting above results we have set $\Gamma_2 = \Gamma_4 = \Gamma_6 = \Gamma$. We note that three velocities $v_g^P$, $v_g^S$, and $v_g^T$ can be made both small and equal by properly adjusting the Rabi frequencies and detunings (see the example given below).

Significant three-body interaction is a key ingredient for the production of three-way entanglement and construction of three-qubit QPG. In our system, such interaction is realized by the giant CPM effect, in which an optical field acquires a large phase shift conditional to the state of the other two optical fields. A three-qubit QPG can be represented by the input-output relations $|\alpha\rangle_P|\beta\rangle_S|\gamma\rangle_T \to \exp(i\phi_{\alpha\beta\gamma})|\alpha\rangle_P|\beta\rangle_S|\gamma\rangle_T$ where $\alpha, \beta, \gamma = 0, 1$ denote three-qubit basis.

We choose two orthogonal light polarizations $|\sigma^-\rangle$ and $|\sigma^+\rangle$ to encode binary information for each qubit. We assume the six-level system shown in Fig. 1 is implemented only when the probe, signal, and trigger all have $\sigma^+$ polarization. For a $\sigma^-$ polarized probe there is no sufficiently close excited state to which level $|1\rangle$ couples and no population in $|3\rangle$ and $|5\rangle$ to drive the signal and trigger transitions. So the probe, signal, and trigger only acquire the trivial vacuum phase shift $\phi_0^i = k_i L$ (i=P, S, T; L denotes the length of the medium). When the probe and signal are $\sigma^+$ and $\sigma^-$ polarized, the probe, subject to the EIT produced by the $|1\rangle - |2\rangle - |3\rangle$ levels $\Lambda$ configuration, acquires a linear phase shift $\phi_\Lambda^P = k_P L(1+2\pi\chi_P^{(1)})$, while the signal and trigger acquire again the vacuum shifts $\phi_0^S$ and $\phi_0^T$. For a $\sigma^+$, $\sigma^+$ and $\sigma^-$ polarized probe, signal and trigger, the first two fields will acquire nonlinear cross-phase shifts $\phi_{3-order}^P$ and $\phi_{3-order}^T$ containing a third-order nonlinear effect, while the last acquire still the vacuum shift $\phi_0^T$. Only when all three pulses have the "right" polarization, they acquire nonlinear cross-phase shifts $\phi_{5-order}^P$, $\phi_{5-order}^S$ and $\phi_{5-order}^T$ containing both three- and

fifth-order nonlinear effects.

Assuming that the input probe, signal, and trigger polarized single photon wave packets can be expressed as a superposition of the circularly polarized states[17, 18, 19], i.e. $|\psi_i\rangle = 1/\sqrt{2}|\sigma^-\rangle_i + 1/\sqrt{2}|\sigma^+\rangle_i$ $(i = P, S, T)$, where $|\sigma^\pm\rangle_i = \int d\omega\xi_i(\omega)a^\dagger_\pm(\omega)|0\rangle$ with $\xi_i(\omega)$ being a Gaussian frequency distribution of incident wave packets centered at frequency $\omega_i$. The photon field operators undergo a transformation while propagating through the atomic medium of length $L$, i.e. $a_\pm(\omega) \to a_\pm(\omega)\exp\{i\omega/c\int_0^L dz n_\pm(\omega, z)\}$. Assuming that $n_\pm(\omega, z)$ (the real part of the refractive index) varies slowly over the bandwidth of the wave packet centered at $\omega_i$, one gets $|\sigma^\pm\rangle_i \to \exp{(-i\phi^i_\pm)}|\sigma^\pm\rangle_i$, with $\phi^i_\pm = \omega/c\int_0^L dz n_\pm(\omega_i, z)$. Thus, the truth table for a polarization three-qubit QPG using our configuration reads:

$$|\sigma^-\rangle_P|\sigma^\pm\rangle_S|\sigma^\pm\rangle_T \to \exp{[-i(\phi^P_0 + \phi^S_0 + \phi^T_0)]}|\sigma^-\rangle_P|\sigma^\pm\rangle_S|\sigma^\pm\rangle_T, \tag{5a}$$

$$|\sigma^+\rangle_P|\sigma^-\rangle_S|\sigma^\pm\rangle_T \to \exp{[-i(\phi^P_\Lambda + \phi^S_0 + \phi^T_0)]}|\sigma^+\rangle_P|\sigma^-\rangle_S|\sigma^\pm\rangle_T, \tag{5b}$$

$$|\sigma^+\rangle_P|\sigma^+\rangle_S|\sigma^-\rangle_T \to \exp{[-i(\phi^P_{3-order} + \phi^S_{3-order} + \phi^T_0)]}|\sigma^+\rangle_P|\sigma^+\rangle_S|\sigma^-\rangle_T, \tag{5c}$$

$$|\sigma^+\rangle_P|\sigma^+\rangle_S|\sigma^+\rangle_T \to \exp{[-i(\phi^P_{5-order} + \phi^S_{5-order} + \phi^T_{5-order})]}|\sigma^+\rangle_P|\sigma^+\rangle_S|\sigma^+\rangle_T. \tag{5d}$$

with $\phi^P_{3-order} = \phi^P_\Lambda + \phi^P_{PS}$, $\phi^S_{3-order} = \phi^S_0 + \phi^S_{SP}$, $\phi^P_{5-order} = \phi^P_\Lambda + \phi^P_{PS} + \phi^P_{PT} + \phi^P_{PST}$, $\phi^S_{5-order} = \phi^S_0 + \phi^S_{ST} + \phi^S_{SPT}$, and $\phi^T_{5-order} = \phi^T_0 + \phi^T_{TPS}$. Explicitly, they are given by

$$\phi^P_{PS} = k_P L\frac{\pi^{3/2}\hbar^2|\Omega_S|^2}{4|D_{34}|^2}\mathrm{Re}[\chi^{(3)}_{PS}]\frac{\mathrm{erf}(\xi_{PS})}{\xi_{PS}}, \tag{6a}$$

$$\phi^P_{PT} = k_P L\frac{\pi^{3/2}\hbar^2|\Omega_T|^2}{4|D_{56}|^2}\mathrm{Re}[\chi^{(3)}_{PT}]\frac{\mathrm{erf}(\xi_{PT})}{\xi_{PT}}, \tag{6b}$$

$$\phi^P_{PST} = k_P L\frac{\pi^{3/2}\hbar^4|\Omega_S|^2|\Omega_T|^2}{4|D_{34}|^2|D_{56}|^2}\mathrm{Re}[\chi^{(5)}_{PST}]\frac{\mathrm{erf}(\xi_{PST})}{\xi_{PST}}, \tag{6c}$$

$$\phi^S_{ST} = k_S L\frac{\pi^{3/2}\hbar^2|\Omega_T|^2}{4|D_{56}|^2}\mathrm{Re}[\chi^{(3)}_{ST}]\frac{\mathrm{erf}(\xi_{ST})}{\xi_{ST}}, \tag{6d}$$

$$\phi^S_{SPT} = k_S L\frac{\pi^{3/2}\hbar^4|\Omega_P|^2|\Omega_T|^2}{4|D_{12}|^2|D_{56}|^2}\mathrm{Re}[\chi^{(5)}_{SPT}]\frac{\mathrm{erf}(\xi_{SPT})}{\xi_{SPT}}, \tag{6e}$$

$$\phi^T_{TPS} = k_T L\frac{\pi^{3/2}\hbar^4|\Omega_P|^2|\Omega_S|^2}{4|D_{12}|^2|D_{34}|^2}\mathrm{Re}[\chi^{(5)}_{TPS}]\frac{\mathrm{erf}(\xi_{TPS})}{\xi_{TPS}}. \tag{6f}$$

where $\xi_{Pi} = \sqrt{2}L(1 - v^P_g/v^i_g)/(\tau_i v^P_g)$ (i=S, T) and $\xi_{ijk} = \sqrt{2}L[(1 - v^i_g/v^j_g)^2/\tau^2_j v^{i2}_g + (1 - v^i_g/v^k_g)^2/\tau^2_k v^{i2}_g]^{1/2}$ (i, j, k=S, P, T) with $\tau_i$ being the time duration of the pulse. If the group velocity matching is satisfied, i.e. $\xi \to 0$, the $\mathrm{erf}[\xi]/\xi$ reaches the maximum value $2/\sqrt{\pi}$.

A three-way entanglement can be calculated by "residual entanglement", which indicates the amount of entanglement among the probe, signal and trigger that cannot be accounted

for by the entanglements of arbitrary two weak fields. As in Ref. [28], the residual entanglement for a three-qubit pure state can be written as follows:

$$\zeta_{PST} = \mathcal{C}^2_{P(ST)} - \mathcal{C}^2_{PS} - \mathcal{C}^2_{PT} = 2(\lambda_1^{PS}\lambda_2^{PS} + \lambda_1^{PT}\lambda_2^{PT}), \tag{7}$$

where $\lambda_1^{PS}$ and $\lambda_2^{PS}$ are respectively the square roots of two eigenvalues of $\rho_{PS}\tilde{\rho}_{PS}$, $\lambda_1^{PT}$ and $\lambda_2^{PT}$ are also defined in a similar way. The reduced density matrix $\rho_{PS} = \mathrm{Tr}_T(\rho_{PST})$ with $\rho_{PST}$ being the density matrix of the output state, and $\tilde{\rho}_{PS} = \sigma_y^P \otimes \sigma_y^S \rho^*_{PS} \sigma_y^P \otimes \sigma_y^S$ with $\sigma_y$ being the $y$-component of the Pauli matrix.

We now consider a practical system working with ultra-cold $^{87}$Rb atomic gas, in which Doppler effect is made small. Atoms are confined in a magneto-optical trap, where the pertinent lower and upper levels are $5S_{1/2}$, $F_L = 1$ , and $5P_{1/2}$, $F_U = 2$. The Zeeman shift of the sublevels in the lower and upper level can be adjusted by the intensity of an applied magnetic field. We take $\delta_1 = \delta_2 = 40.0 \times 10^7$ s$^{-1}$, $\delta_3 = \delta_4 = -40.0 \times 10^7$ s$^{-1}$ ($\delta_{12} = \delta_{14} = 0$), $\delta_5 = 2.5 \times 10^7$ s$^{-1}$ ($\delta_5 \gg \Gamma/2$ should be satisfied to ensure a small absorption), $\Omega_C = 8.0\times 10^7$ s$^{-1}$, $\Omega_B = 5.2\times 10^7$ s$^{-1}$, $\Omega_P = 2.4\times 10^7$ s$^{-1}$, $\Omega_S = 2.5\times 10^7$ s$^{-1}$, $\Omega_T = 1.4\times 10^7$ s$^{-1}$, $\Gamma = 0.5\times 10^7$ s$^{-1}$, and $N_a = 10^{12}$ cm$^{-3}$. The probe, signal and trigger have a mean amplitude of about one photon when the beams are tightly focused and has a time duration about one microsecond. With the given parameters, one recognize that the system remains only the fifth-order susceptibilities and acquire the nontrivial nonlinear phase shifts entirely caused by the fifth-order nonlinearity only when all weak fields have the "right" polarization. Based on which, the pase-gating mechanism is presented. The group velocities of the weak fields read $v_g^P \simeq 5.5 \times 10^3$ m/s, $v_g^T \simeq 6.0 \times 10^3$ m/s, and $v_g^T \simeq 8.1 \times 10^3$ m/s. A total nonlinear phase shift of $5\pi$ radians can be obtained for $L \simeq 0.095$cm, and the residual entanglement $\zeta_{PST} \simeq 25\%$. The imaginary part of the fifth-order susceptibilities is one order of magnitude smaller than the real part, and hence be neglected safely.

With the above parameters, we realize an operation $\hat{U} = |000\rangle\langle 000| + |001\rangle\langle 001| + |010\rangle\langle 010| + |011\rangle\langle 011| + |100\rangle\langle 100| + |101\rangle\langle 101| + |110\rangle\langle 110| - |111\rangle\langle 111|$. By applying a single qubit rotation $\hat{R}_i$ to the trigger field where

$$\hat{R}_i(\theta,\varphi) = \begin{pmatrix} \cos\dfrac{\theta}{2} & ie^{-i\varphi}\sin\dfrac{\theta}{2} \\ -ie^{i\varphi}\sin\dfrac{\theta}{2} & -\cos\dfrac{\theta}{2} \end{pmatrix}, \tag{8}$$

we can easily obtain the Toffoli gate by $\hat{U}_{\text{Toffoli}} = \hat{R}_T(\pi/2,\pi,2)\hat{U}\hat{R}_T^{-1}(\pi/2,\pi,2)$. The explicit operation is illustrated in Fig. 2.

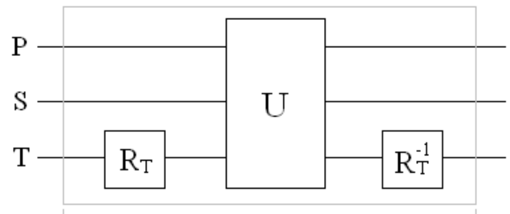


FIG. 2: The quantum circuit for realizing the Toffoli gate.

To sum up, we have investigated the nonlinear optical response of a six-level atomic system under a configuration of electromagnetically induced transparency. The resultant giant fifth-order nonlinearity and vanishing linearity and third-order nonlinearity provided by the system can produce efficient three-way entanglement among the weak probe, signal, and trigger laser pulses. Unlike [21], here we have addressed a feasible method to satisfy the group velocity matching among three optical pulses without using isotopes or solid quantum dots. In addition, we have studied the possibility of implementing a robust three-qubit QPG, which can be further transferred to a Toffoli gate by applying a single qubit rotation. The practical realization of such a six-level system is easily achievable in a alkali atomic system in a gas cell. The results provided in this work may be useful for guiding experimental realization of three-way entanglement and three-qubit phase gates and facilitating practical applications in quantum information and computation.

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